

## Mock JEE Main – 5 (CBT) | JEE - 2024

Date: 20/1/2024

Maximum Marks: 300

Timing: 9:30 AM to 12:30 PM

Duration : 3.0 Hours

### General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300**.
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices **(A), (B), (C)** and **(D)**, out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **10 Numerical Value Type Questions** Out of which **ONLY 5 (any)** questions have to be attempted. You will **NOT** be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.  
The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

### Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Name of the Candidate (In CAPITALS) : .....

Roll Number : .....

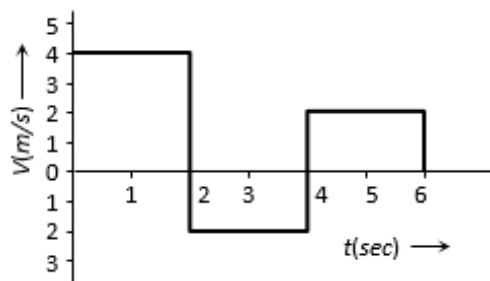
OMR Bar Code Number : .....

Candidate's Signature : ..... Invigilator's Signature .....

**SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

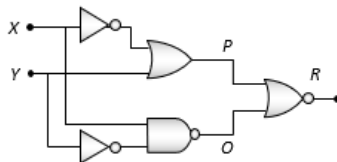
- The electric field for an electromagnetic wave in free space is  $E = 30 \cos(kz - 5 \times 10^8 t)$ , where magnitude of  $E$  is in V/m. The magnitude of ' $k$ ' is (velocity of EM wave in free space  $= 3 \times 10^8 \text{ m/s}$ )  
 (A)  $0.46 \text{ rad m}^{-1}$  (B)  $3 \text{ rad m}^{-1}$  (C)  $1.66 \text{ rad m}^{-1}$  (D)  $0.83 \text{ rad m}^{-1}$
- The velocity-time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 0 to 6 second, are respectively:



- (A) 8 m, 16 m (B) 16 m, 8 m (C) 16 m, 16 m (D) 8 m, 8 m
- Two planets of radii in the ratio 2 : 3 are made from the material of density in the ratio 3 : 2. Then the ratio of acceleration due to gravity ( $g_1 / g_2$ ) at the surface of the two planets will be:  
 (A) 1 (B) 2.25 (C) 4/9 (D) 0.12
- Assuming the coefficient of friction between the road and tyres of a car to be 0.5, the maximum speed with which the car can move round a curve of 40.0 m radius without slipping, if the road is unbanked, should be :  
 (A) 25 m/s (B) 19 m/s (C) 14 m/s (D) 11 m/s

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5. Figure gives a system of logic gates. From the study of truth table it can be found that to produce a high output (1) at  $R$ , we must have:



- (A)  $X = 0, Y = 1$  (B)  $X = 1, Y = 1$  (C)  $X = 1, Y = 0$  (D)  $X = 0, Y = 0$
6. A parallel plate capacitor has plate of length ' $l$ ', width ' $w$ ' and separation of plates is ' $d$ '. It is connected to a battery of emf  $V$ . A dielectric slab of the same thickness ' $d$ ' and of dielectric constant  $k = 4$  is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?
- (A)  $\frac{l}{3}$  (B)  $\frac{l}{4}$  (C)  $\frac{2l}{3}$  (D)  $\frac{l}{2}$
7. Two bulbs of 500 watt and 200 watt are manufactured to operate on 220 volt line. The ratio of heat produced in 500 W and 200 W bulbs, in two cases, when firstly they are joined in parallel and secondly in series, respectively, will be:
- (A)  $\frac{5}{2}, \frac{2}{5}$  (B)  $\frac{5}{2}, \frac{5}{2}$  (C)  $\frac{2}{5}, \frac{5}{2}$  (D)  $\frac{2}{5}, \frac{2}{5}$
8. A conducting circular loop is placed in a uniform magnetic field of induction  $B$  tesla with its plane normal to the field. Now, the radius of the loop starts shrinking at the rate  $\left(\frac{dr}{dt}\right)$ . Then, the magnitude of induced emf in the loop at the instant when the radius is  $r$ , is:
- (A)  $\pi r B \left(\frac{dr}{dt}\right)$  (B)  $2\pi r B \left(\frac{dr}{dt}\right)$  (C)  $\pi r^2 \frac{dB}{dt}$  (D)  $\left(\frac{\pi r^2}{2}\right)^2 B \left(\frac{dr}{dt}\right)$

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9. One kg of water, at  $20^{\circ}\text{C}$ , heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of  $20\Omega$ . The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to: [Specific heat of water =  $4200 \text{ J/(kg }^{\circ}\text{C)}$ , Latent heat of water =  $2260 \text{ kJ/kg}$ ]  
(A) 3 minutes (B) 16 minutes (C) 22 minutes (D) 10 minutes
10. During an adiabatic expansion of 2 moles of a gas, the change in internal energy was found to be equal to  $-50\text{J}$ . The work done during the process is:  
(A) Zero (B)  $100\text{J}$  (C)  $-50\text{J}$  (D)  $50\text{J}$
11. Refractive index of glass is  $\frac{3}{2}$  and refractive index of water is  $\frac{4}{3}$ . If the speed of light in glass is  $2.00 \times 10^8 \text{ m/s}$ , the speed in water will be:  
(A)  $2.67 \times 10^8 \text{ m/s}$  (B)  $2.25 \times 10^8 \text{ m/s}$  (C)  $1.78 \times 10^8 \text{ m/s}$  (D)  $1.50 \times 10^8 \text{ m/s}$
12. Threshold wavelength for photoelectric effect on sodium is  $5000 \text{ \AA}$ . Its work function is:  
(A)  $15 \text{ J}$  (B)  $16 \times 10^{-19} \text{ J}$  (C)  $4 \times 10^{-19} \text{ J}$  (D)  $4 \times 10^{-20} \text{ J}$
13. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the  $+x$  - direction and a magnetic field along the  $+z$  direction, then:  
(A) Positive ions deflect towards  $+y$  - direction and negative ions towards  $-y$  - direction  
(B) All ions deflect towards  $+y$  - direction  
(C) All ions deflect towards  $-y$  -direction  
(D) Positive ions deflect towards  $-y$  - direction and negative ions towards  $+y$  - direction

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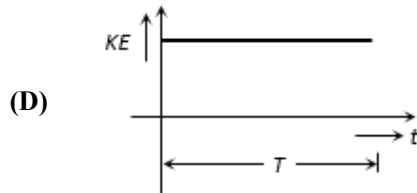
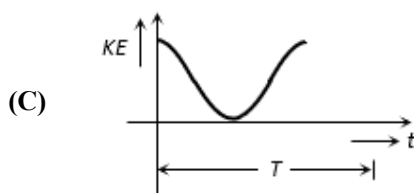
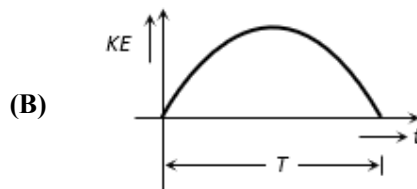
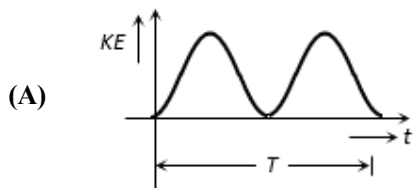
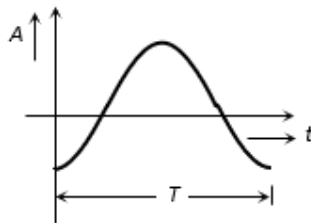
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14. The relation between voltage sensitivity ( $\sigma_V$ ) and current sensitivity ( $\sigma_i$ ) of a moving coil galvanometer is: (Resistance of Galvanometer =  $G$ )

(A)  $\frac{\sigma_i}{G} = \sigma_V$       (B)  $\frac{\sigma_V}{G} = \sigma_i$       (C)  $\frac{G}{\sigma_V} = \sigma_i$       (D)  $\frac{G}{\sigma_i} = \sigma_V$

15. Graph between acceleration  $A$  and time  $t$  of a body in S.H.M. is given by a curve shown below. Then corresponding graph, between kinetic energy (K.E.) and time  $t$  is correctly represented by:



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16. If temperature scale is changed from  $^{\circ}C$  to  $^{\circ}F$ , the numerical value of specific heat will:  
 (A) Increases (B) Decreased  
 (C) Remains unchanged (D) May increase or decrease
17. Choose any one of the following four responses:  
**Statement 1 :** 'Light year' and 'Angstrom' both measure distance.  
**Statement 2 :** Light year and year, both measure time.  
 (A) Both Statements 1 and 2 are correct.  
 (B) Statement 1 is correct but statement 2 is incorrect.  
 (C) Both statements 1 and 2 are incorrect  
 (D) Statement 1 is incorrect but Statement 2 is Correct
18. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from  $\tau_1$  to  $\tau_2$ . If  $\frac{C_p}{C_v} = \gamma$  for this gas then a good estimate for  $\frac{\tau_2}{\tau_1}$  is given by:  
 (A)  $\frac{1}{2}$  (B)  $2^{\left(\frac{\gamma+1}{2}\right)}$  (C)  $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$  (D)  $\left(\frac{1}{2}\right)^{\gamma}$
19. The average force necessary to stop a bullet of mass 20 g moving with a speed of 250 m/s, as it penetrates into the wood for a distance of 12 cm is:  
 (A)  $2.2 \times 10^3 N$  (B)  $3.2 \times 10^3 N$  (C)  $4.2 \times 10^3 N$  (D)  $5.2 \times 10^3 N$
20. A proton has kinetic energy  $E = 100 \text{ KeV}$  which is equal to that of a photon. The wavelength of photon is  $\lambda_2$  and that of proton is  $\lambda_1$ . The ratio of  $\lambda_2 / \lambda_1$  is proportional to:  
 (A)  $E^2$  (B)  $E^{-1/2}$  (C)  $E^{-1}$  (D)  $E^{1/2}$

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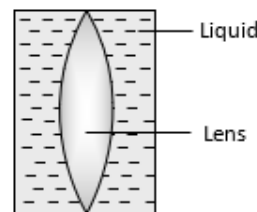
## SECTION-2

**Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted.** The answer to each question should be **rounded off to the nearest integer**.

1. An electron jumps from the 4<sup>th</sup> orbit to the 2<sup>nd</sup> orbit of hydrogen atom. Given the Rydberg's constant

$R = 10^5 \text{ cm}^{-1}$ . The frequency in  $\text{Hz}$  of the emitted radiation will be  $\frac{x}{16} \times 10^{15} \text{ Hz}$ ,  $x$  will be \_\_\_\_\_.

2. Shown in the figure here is a convergent lens placed inside a liquid. The lens has focal length  $+20 \text{ cm}$  when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, the magnitude of focal length of the system of lens + liquid is \_\_\_\_\_  $\text{cm}$ .



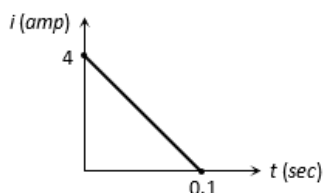
3. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is \_\_\_\_\_  $\text{Hz}$ .

4. A force  $F = (5\hat{i} + 3\hat{j})$  newton is applied over a particle which displaces it from origin to the point  $r = (2\hat{i} - 1\hat{j})$  metres. The work done on the particle is \_\_\_\_\_ joules.

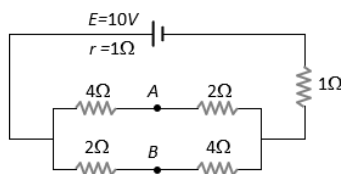
5. A circular disc of radius  $R$  carries surface charge density  $\sigma(r) = \sigma_0 \left(1 - \frac{r}{R}\right)$ , where  $\sigma_0$  is a constant and  $r$  is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is  $\phi_0$ . Electric flux through another spherical surface of radius  $\frac{R}{4}$  and concentric with the disc is  $\phi$ . Then  $10 \frac{\phi_0}{\phi}$  is \_\_\_\_\_.

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6. The moment of inertia of a circular disc of radius 2 m and mass 2 kg, about an axis passing through its centre of mass is  $2kg-m^2$ . Its moment of inertia about an axis parallel to this axis and passing through its edge (in  $kg-m^2$ ) is \_\_\_\_\_.
7. Two wires of copper having the length in the ratio 4 : 1 and their radii ratio as 1 : 4 are stretched by the same force. The ratio of longitudinal strain in the two will be  $x:1$ ,  $x$  is \_\_\_\_\_.
8. Some magnetic flux is changed in a coil of resistance 10 ohm. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in webers is \_\_\_\_\_.



9. A ball is thrown at an angle of  $60^\circ$  to the horizontal. It falls on the ground at a horizontal distance of 50 m. If the ball is thrown with double the velocity at the same angle, it will fall on the ground at a distance (in meters) of \_\_\_\_\_.
10. In the circuit shown below, the cell has an e.m.f. of 10 V and internal resistance of 1 ohm. The other resistances are shown in the figure. The potential difference  $V_B - V_A$  is \_\_\_\_\_ (in volts).



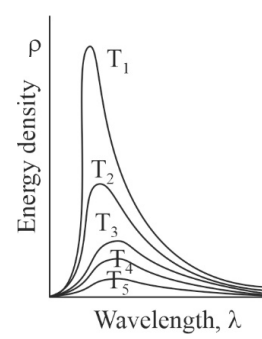
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**SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

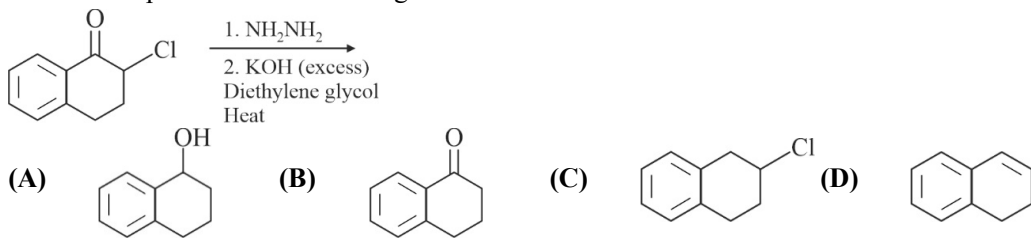
- Among the following, the correct statement for thionyl tetrafluoride is:
  - The geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond on the trigonal plane
  - The geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond perpendicular to the trigonal plane
  - The geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond on the square plane
  - The geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond perpendicular to the square plane
- The plots of energy density (energy per unit area) vs wavelength for blackbody radiation at various temperatures is given below.  
The correct option among the following is:
  - $T_1 > T_2 > T_3 > T_4 > T_5$
  - As temperature increases, the wavelength at which the intensity is maximum shifts towards the higher energy regions of the electromagnetic spectrum
  - Radiations of all wave lengths are emitted, absorbed, reflected, and refracted by the black body
  - Black body is in thermal equilibrium with its surroundings



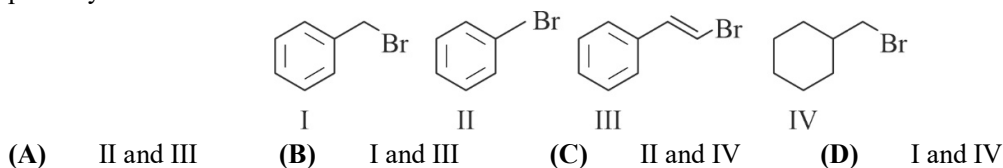
(A) (i) and (ii)      (B) (ii) and (iii)      (C) (i), (iii) and (iv)      (D) (i), (ii) and (iv)

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3. Find out the product in the following reaction.



4. Which of the following compounds will NOT undergo the Finkelstein reaction with NaI via  $S_N2$  pathway?



5. The correct IUPAC name of potassium permanganate is:

(A) Potassium tetraoxomanganate (VI) (B) Potassium tetraoxidopermanganate (VII)  
 (C) Potassium tetraoxidomanganese (VII) (D) Potassium tetraoxidomanganate (VII)

6. The order of screening effect of electrons of s, p, d and f orbitals of a given shell of an atom on its outer shell electrons is:

(A)  $s > p > d > f$  (B)  $f > d > p > s$  (C)  $p < d < s > f$  (D)  $f > p > s > d$

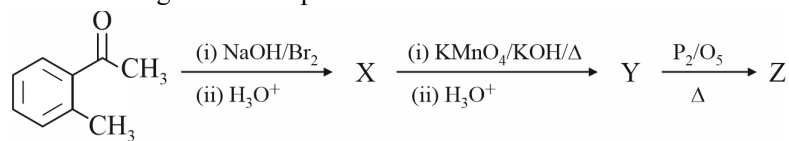
7. Which of the following is amphoteric oxide?

$Mn_2O_7, CrO_3, Cr_2O_3, CrO, V_2O_5, V_2O_4$

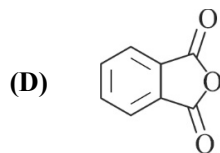
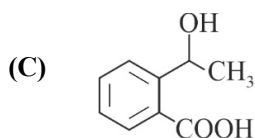
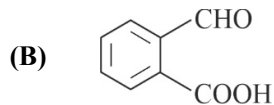
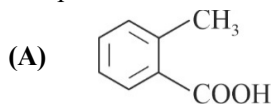
(A)  $V_2O_5, Cr_2O_3$  (B)  $Mn_2O_7, CrO_3$  (C)  $CrO, V_2O_5$  (D)  $V_2O_5, V_2O_4$

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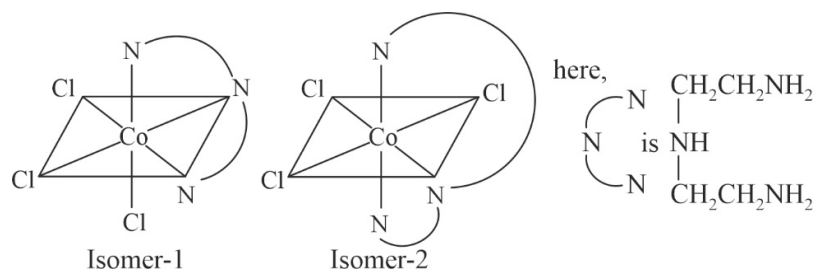
8. Consider the following reaction sequence:



The product Z is:



9. Two isomers of  $[\text{CoCl}_3(\text{NH}_2\text{CH}_2\text{CH}_2\text{NHCH}_2\text{CH}_2\text{NH}_2)]$  are shown below:

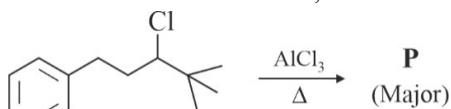


The correct statement about Isomer-1 and Isomer-2 is:

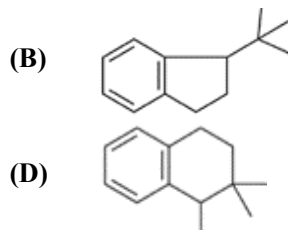
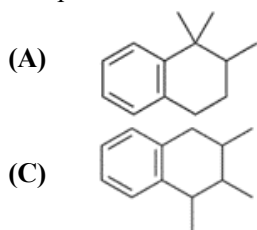
- (A) Isomer-1 and Isomer-2 are facial and meridional, respectively  
 (B) In Isomer-1 and Isomer-2, all Cl-Co-Cl and N-Co-N bond angles are  $90^\circ$   
 (C) Neutral aqueous solution of both these isomers will react with  $\text{Ag}^+$  ion to give  $\text{AgCl}$   
 (D) In Isomer-2, two Cl-Co-Cl bond angles are  $180^\circ$

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10. In the reaction shown below,



The product P is:



11. The complementary DNA sequence of the given DNA 5'-G-A-A-T-T-C-3' is:

- (A) 5'-C-T-T-A-A-G-3'      (B) 5'-C-U-U-A-A-G-3'  
 (C) 3'-C-T-T-A-A-G-5'      (D) 3'-G-A-A-T-T-C-5'

12. **Assertion :** Actinoids form relatively less stable complexes as compared to lanthanoids.

**Reason :** Actinoids can utilise their 5f orbitals along with 6d orbitals in bonding but lanthanoids do not use their 4f orbital for bonding.

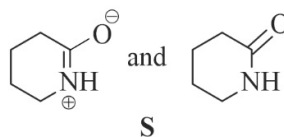
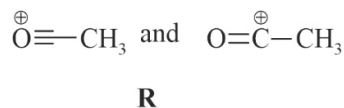
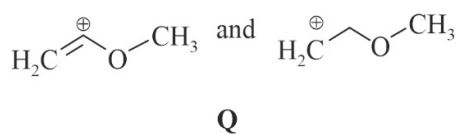
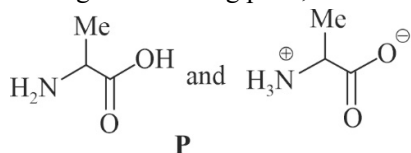
- (A) Both assertion and reason are true, and reason is the correct explanation of the assertion  
 (B) Bioth assertion and reason are true but reason is not the correct explanation of assertion  
 (C) Assertion is not true but reason is true  
 (D) Both assertion and reason are false

13. Electrolysis of aqueous  $\text{CuSO}_4$  (0.1M) was carried out in two cells I and II. In I, the electrodes are of Cu and in II they were of Pt. As the electrolysis proceeds pH of the electrolyte solution will:

- (A) Decrease in II and remain the same in I      (B) Remain the same in both I and II  
 (C) Increase in both I and II      (D) Increase in I and decrease in II

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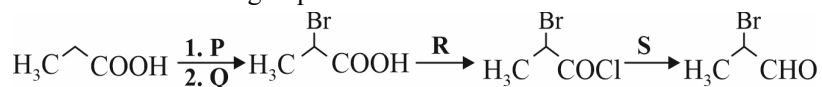
14. Among the following pairs,



The pairs that represent resonance structures are:

- (A) P, Q and R      (B) Q, R and S      (C) Q and S      (D) R and S

15. Consider the following sequence of reactions.



The correct reagents (P, Q, R and S) required are:

- (A) **P** =  $\text{Br}_2$ /red phosphorous; **Q** =  $\text{H}_2\text{O}$ ; **R** =  $\text{SOCl}_2$ ; **S** =  $\text{H}_2$ , Pd-BaSO<sub>4</sub>  
 (B) **P** =  $\text{Br}_2$ /red phosphorous; **Q** =  $\text{H}_3\text{O}^+$ ; **R** =  $\text{SOCl}_2$ ; **S** =  $\text{LiAlH}_4$   
 (C) **P** =  $\text{Br}_2$ /NaOH; **Q** =  $\text{H}_2\text{O}$ ; **R** =  $\text{PCl}_3$ ; **S** = DIBAL-H  
 (D) **P** =  $\text{PBr}_3$ ; **Q** =  $\text{H}_3\text{O}^+$ ; **R** =  $\text{Cl}_2$  /  $\text{FeCl}_3$ ; **S** = Pd- BaSO<sub>4</sub>

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16. The correct statement regarding the free energy change ( $\Delta G$ ) and the standard free energy change ( $\Delta G^0$ ) for (i) melting of ice and (ii) a chemical reaction  $P \rightarrow Q$  is:
- (A) The process of slow melting of ice at  $0^\circ\text{C}$  and atmospheric pressure is associated with a negative value of  $\Delta G$
- (B) The process of slow melting of ice at  $0^\circ\text{C}$  at atmospheric pressure is associated with a positive value of  $\Delta G$
- (C) The reaction  $P \rightarrow Q$  with a positive value of  $\Delta G^0$  can never be spontaneous
- (D) The reaction  $P \rightarrow Q$  with a positive value of  $\Delta G^0$  can be spontaneous, but the reaction will reach equilibrium with  $[Q]_{\text{eq}} < [P]_{\text{eq}}$ .
17. Choose the correct statement among the following:
- (A) N – N single bond is stronger than P – P single bond
- (B) Anomalous behaviour of fluorine is due to its small size, highest electronegativity, low F – F bond dissociation enthalpy and non-availability of d-orbitals in valence shell
- (C) Metalloids exists in p & d-block
- (D)  $p\pi - p\pi$  bonding is strong for heavier elements
18. The number of isomeric alkenes with molecular formula  $\text{C}_5\text{H}_{10}$  is: (Taking stereoisomers into account)
- (A) 4                      (B) 5                      (C) 6                      (D) 7

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19. Initial concentrations of the reactants and the corresponding half-lives for the reaction  $P + Q \rightarrow R$  are given below. The rate law for the reaction is:

Entry	$[P_0]$ ( $\text{mol dm}^{-3} \times 10^{-6}$ )	$[Q_0]$ ( $\text{mol dm}^{-3} \times 10^{-6}$ )	$t_{1/2}$ (s)
1	500	10	30
2	500	20	60
3	10	500	60
4	20	500	60

- (A)  $dR / dt = k[P]$  (B)  $dR / dt = k[P]/[Q]$   
 (C)  $dR / dt = k[Q]$  (D)  $-d[P] / dt = k[P]/[Q]$
20.  $\text{Br}_2$  disproportionate to  $\text{Br}^-$  and  $\text{BrO}_3^-$  in a hot alkaline solution as



The equivalent weight of  $\text{Br}_2$  is: ( $M$  = molar mass of  $\text{Br}_2$ )

- (A)  $\frac{M}{5}$  (B)  $\frac{M}{6}$  (C)  $\frac{3M}{5}$  (D)  $\frac{5M}{3}$

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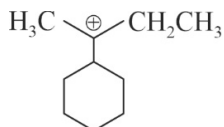
**SECTION-2**

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

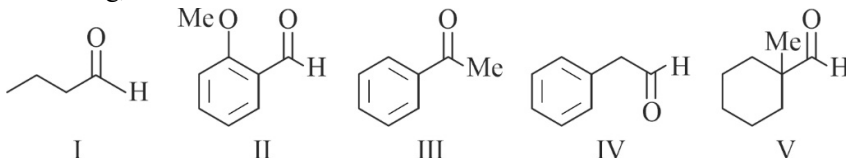
1. The standard e.m.f of a galvanic cell involving cell reaction with  $n = 2$  is found to be 0.295 V at 25°C. The equilibrium constant of the reaction is  $y \times 10^{10}$ . The value of  $y$  is \_\_\_\_\_.

[Use  $\frac{2.303RT}{F} = 0.0590$ ]

2. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is \_\_\_\_\_.



3. Consider the reaction,  $P(aq) \rightleftharpoons Q(aq)$  with an equilibrium constant  $K = 1.5$ . The reaction is started in a vessel with a concentration of  $[P]$  of 2 M and concentration of  $[Q] = 0$ . When the equilibrium is established, half the amount of P is removed, and the reaction is allowed to re-equilibrate. The concentration of Q in the vessel (in M) is  $x \times 10^{-2}$ . Find  $x$ ?
4. The number of moles of hydrogen molecules required to produce 20 moles of ammonia through Haber's process is \_\_\_\_\_.
5. Among the following,

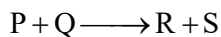


The number of compounds which can be reduced with formaldehyde and conc. Aq. KOH, are \_\_\_\_\_.

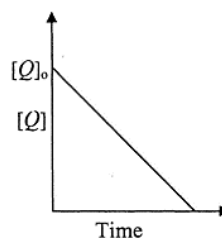
**SPACE FOR ROUGH WORK**



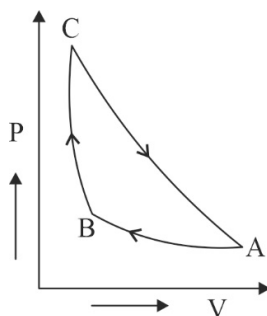
6. In the reaction,



The time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is \_\_\_\_\_.



7. The number of geometric isomers possible for the complex  $[\text{CoL}_2\text{Cl}_2]^-$  ( $\text{L} = \text{H}_2\text{NCH}_2\text{CH}_2\text{O}^-$ ).
8. The figure represents the process AB, BC and CA undertaken by a certain mass of an ideal gas. Along the path AB, the gas is isothermally compressed with release of 800J heat to the surroundings. It is then compressed adiabatically along the path BC and the work done is 500J. The gas then returns to the state A along path CA and absorbs 100 J heat from the surroundings. The magnitude of work done by the gas along the path CA is \_\_\_\_\_.



9. A 0.004 M solution of  $\text{K}_2\text{SO}_4$  is isotonic with a 0.010 M solution of glucose at the same temperature. The apparent percent degree of dissociation of  $\text{K}_2\text{SO}_4$  is \_\_\_\_\_.
10. Chlorination of propane gives four dichloro products. One of them is optically active. The number of trichloro products possible from the optically active dichloro product is (excluding stereoisomers):

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## PART III: MATHEMATICS

MARKS: 100

**SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. If  $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$  then  $\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$  is equal to:

(A)  $A - \frac{1}{2} - \frac{1}{\pi+2}$  (B)  $\frac{1}{2} + \frac{1}{\pi+2} - A$  (C)  $\frac{1}{\pi+2} - A$  (D)  $1 + \frac{1}{\pi+2} - A$

2. Let  $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$  and  $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$  be two vectors, such that  $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$ . Then the length of the projection of  $\vec{b} - 2\vec{a}$  on  $\vec{b} + \vec{a}$  is equal to:

(A) 2 (B)  $\frac{39}{5}$  (C) 9 (D)  $\frac{46}{5}$

3. Let the line  $y = mx$  and the ellipse  $4x^2 + 9y^2 = 1$  intersect at point  $P$  in the first quadrant. If the normal to the ellipse at  $P$  meets the coordinate axes at  $A(\alpha, 0)$  and  $B\left(0, -\frac{5}{8\sqrt{3}}\right)$ , then area of triangle  $OAB$  is equal to : (where  $O$  is origin)

(A)  $\frac{1}{16\sqrt{3}}$  (B)  $\frac{25}{576\sqrt{3}}$  (C)  $\frac{9}{144\sqrt{3}}$  (D)  $\frac{25}{288\sqrt{3}}$

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4. Let the mean and standard deviation of marks of class  $A$  of 100 students be respectively 40 and  $\alpha (> 0)$ , and the mean and standard deviation of marks of class  $B$  of  $n$  students be respectively 55 and  $30 - \alpha$ . If the mean and variance of the marks of the combined class of  $100 + n$  students are respectively 50 and 350, then the sum of variances of classes  $A$  and  $B$  is:  
 (A) 500 (B) 650 (C) 450 (D) 900
5. Let  $f(x) = [x] + \sqrt{\{x\}}$ . (Here  $[.]$  is integral part of ' $x$ ' and  $\{.\}$  fractional part of ' $x$ ' functions) then which of the following is **NOT** correct for  $f(x)$ .  
 (A) Continuous in  $(-2, 2)$  (B) Non differentiable at 3 points in  $(-2, 2)$   
 (C) Many one in  $(-2, 2)$  (D) Discontinuous at 2 points in  $(-2, 2)$
6. How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram cannot have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.  
 (A) 440 (B) 450 (C) 540 (D) 550
7. Let two non-collinear unit vectors  $\hat{a}$  and  $\hat{b}$  form an acute angle. A point  $P$  moves so that at any time " $t$ ", the position vector  $\overrightarrow{OP}$  (where ' $O$ ' is the origin) is given by  $\hat{a} \cos t + \hat{b} \sin t$ . When ' $P$ ' is farthest from origin ' $O$ ', let ' $M$ ' be the length of  $\overrightarrow{OP}$  and  $\hat{\mu}$  be the unit vector along  $\overrightarrow{OP}$ , then  
 (A)  $\hat{\mu} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$  (B)  $\hat{\mu} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$   
 (C)  $\hat{\mu} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$  (D)  $\hat{\mu} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

SPACE FOR ROUGH WORK

8. If  $z$  is a complex number such that  $\arg\left(\frac{z-2-i}{z+2-i}\right) = \frac{\pi}{2}$  then locus of  $z$  is:
- (A)  $|z-i|=2$  and  $\text{img}(z) > 1$  (B)  $|z-i|=2$  and  $\text{img}(z) < 1$   
 (C)  $|z-1|=2$  and  $\text{real}(z) > 1$  (D)  $|z+1|=2$  and  $\text{real}(z) < 1$
9. There are 3 clubs  $A$ ,  $B$  and  $C$  in a town with 40, 50 and 60 members. 10 people are members of all three clubs. 70 people are members of exactly one club. A member is randomly selected. Find the probability that he had membership of exactly 2 clubs.
- (A)  $\frac{7}{15}$  (B)  $\frac{1}{6}$  (C)  $\frac{3}{21}$  (D)  $\frac{5}{21}$
10. Let  $A$  be a matrix of order  $2 \times 2$ , whose entries are from the set  $\{0, 1, 2, 3, 4, 5\}$ . Then the probability, if the sum of entries of  $A$  is a prime number  $p$ , ( $2 < p < 8$ ) is:
- (A)  $\frac{180}{6^4}$  (B)  $\frac{195}{6^4}$  (C)  $\frac{175}{6^4}$  (D)  $\frac{199}{6^4}$
11. Circle  $x^2 + y^2 + 8y = 9$  intersects ellipse  $\frac{x^2}{16} + \frac{y^2}{7} = 1$  at points  $A$  and  $B$ . Tangents and normals are drawn to the ellipse at  $A$  &  $B$  such that tangents intersect at  $P$  and normals intersect at  $Q$ . Then  $PQ$  is equal to:
- (A) 3 (B) 4 (C) 5 (D) 10

SPACE FOR ROUGH WORK

12. The number of integral solution of  $(\log_2 x)^4 - \left(\log_{\frac{1}{2}} \frac{x^5}{4}\right)^2 - 20\log_2 x + 148 < 0$  :
- (A) 7 (B) 8 (C) 6 (D) 10
13. The area defined by  $1 \leq |x-2| + |y+1| \leq 2$  is:
- (A) 2 (B) 4 (C) 6 (D) 8
14. For any two real numbers  $x$  and  $y$ , we define  $xRy$  if and only if  $\sin^2 x + \cos^2 y = 1$ . The relation  $R$  is:
- (A) Symmetric but not transitive (B) Reflexive but not symmetric  
(C) Transitive but not reflexive (D) An equivalence relation
15. The general solution of the Differential Equation  $3x^2y^2 + \cos(xy) - xy \sin(xy) + \frac{dy}{dx} \{2x^3y - x^2 \sin(xy)\} = 0$  is
- (A)  $x(x^2y^2 + \cos(xy)) = k; k \in R$  (B)  $y(x^2y^2 + \cos(xy)) = k; k \in R$   
(C)  $x(x^2y^2 - \cos(xy)) = k; k \in R$  (D)  $y(x^2y^2 - \cos(xy)) = k; k \in R$
16. If  $A$  &  $B$  are square matrices of order 2 such that  $A + \text{adj}(B^T) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  &  $A^T - \text{adj}(B) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ , then:
- (A)  $B$  is symmetric matrix (B)  $A^n = A \forall n \in N$   
(C)  $|A + A^2 + A^3 + A^4 + A^5| \neq 0$  (D)  $|B + B^2 + B^3 + B^4 + B^5| = 0$

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17.  $L_1$  and  $L_2$  are two lines whose vector equations are

$$L_1 : \vec{r} = \lambda((\cos \theta + \sqrt{3})\hat{i} + (\sqrt{2} \sin \theta)\hat{j} + (\cos \theta - \sqrt{3})\hat{k})$$

$$L_2 : \vec{r} = \mu(a\hat{i} + b\hat{j} + c\hat{k}),$$

Where  $\lambda$  and  $\mu$  are scalars and  $\alpha$  is the acute angle between  $L_1$  and  $L_2$ . If the angle ' $\alpha$ ' is independent of  $\theta$  then the value of ' $\alpha$ ' is:

- (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{2}$

18. In a quadrilateral  $ABCD$ ,  $\overrightarrow{AC}$  is the bisector of  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$ , angle between  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$  is  $\frac{2\pi}{3}$ ,  $|\overrightarrow{AC}| = 3$ ,  $|\overrightarrow{AB}| = 5$ ,  $|\overrightarrow{AD}| = 5$ . Then the angle between  $\overrightarrow{BA}$  and  $\overrightarrow{CD}$  is:

- (A)  $\cos^{-1} \frac{1}{\sqrt{7}}$  (B)  $\cos^{-1} \frac{1}{2\sqrt{7}}$  (C)  $\cos^{-1} \frac{2}{\sqrt{7}}$  (D)  $\cos^{-1} \sqrt{\frac{3}{7}}$

19. In  $\triangle ABC$ , if  $\cot \theta = \cot A + \cot B + \cot C$ , then  $\frac{\sin(A-\theta)\sin(B-\theta)\sin(C-\theta)}{\sin^3 \theta}$ :

- (A) 1 (B) -1 (C)  $-\sqrt{2}$  (D)  $\sqrt{2}$

20. If  $a_1, a_2, a_3, \dots, a_n$  are in HP and  $f(K) = \left( \sum_{r=1}^n a_r \right) - a_K$  then  $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \dots, \frac{a_n}{f(n)}$ :

- (A) AP (B) GP (C) HP (D) AGP

SPACE FOR ROUGH WORK

**SECTION-2**

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

- Let  $(1+x+2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$  and  $a_1 + a_3 + a_5 + \dots + a_{37} = 2^\alpha(2^\beta - 21)$ , then  $|\alpha - \beta|$  is equal to \_\_\_\_\_.
- Sum to  $n$  terms of the series  $1 + 5 + 19 + 65 + 211 + \dots = \frac{1}{2}(\alpha^{n+1} - 2^{n+\beta} + 1)$ , then  $\alpha - \beta$  is equal to \_\_\_\_\_.
- If  $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x) \sin x} dx, n = 0, 1, 2, \dots$ , then the least positive value of  $K$  for which  $I_n = I_{n+K}$ .
- Seven people leave their bags outside temple and returning after worshipping picked one bag each at random. In how many ways at least one and at most three of them get their correct bags?
- Let  $L_1 : x = y = z, L_2 : x - 1 = y - 2 = z - 3$  be two lines. Let from origin  $O(0, 0, 0)$  on  $L_1$ , perpendicular is drawn to  $L_2$  has foot  $A$ . Segment  $OA$  is rotated about  $O$  by an angle  $90^\circ$  such that  $L_2$  moves along with it, without changing its direction & becomes  $L_3$ .  $A$  becomes  $B(\alpha, \beta, \gamma)$  then  $\alpha + \beta + \gamma =$
- The sum of the rational terms in the expansion of  $(\sqrt{2} + 3^{1/5})^{10}$  is:

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**SPACE FOR ROUGH WORK**

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7. A function  $f: R \rightarrow R$  satisfies the equation  $f(x+y) = f(x) \cdot f(y)$  for all  $x, y \in R$ ,  $f(x) \neq 0$ , suppose that the function is differentiable at  $x=0$ , and  $f'(0) = 2$ , and  $f'(x) = kf'(x)$  then  $k$  is equal to \_\_\_\_\_.
8. If the radius of director circle of curve represented by the equation  $\frac{x^2}{\sin \sqrt{2} - \sin \sqrt{3}} + \frac{y^2}{\cos \sqrt{2} - \cos \sqrt{3}} = 1$  is ' $r$ ' and  $r^2 = (\sin \sqrt{2} + \cos \sqrt{\alpha}) - (\sin \sqrt{3} + \cos \sqrt{\beta})$  then the value of  $4 + \alpha - \beta =$  \_\_\_\_\_.
9. Consider a cubic polynomial  $f(x) = x^3 + 6x^2 + 4x + 5$  consider the expression  $p_i = \alpha_1^i + \alpha_2^i + \alpha_3^i$  where  $\alpha_1, \alpha_2$  &  $\alpha_3$  are roots of equation,  $i \geq 3$  then  $\left| \frac{p_{10} + 6p_9 + 4p_8}{5p_7} \right|$  is equal to \_\_\_\_\_.
10. Let  $\text{tr}(X)$  and  $\text{adj}(X)$  denote the trace and adjoint of a square matrix  $X$ . If  $M$  is a non-singular square matrix of order 3 such that  $M^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$ , then the value of  $|15\text{tr}(\text{adj}(M))|$  is \_\_\_\_\_.

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